Trigonometry
8e

Charles P. McKeague • Mark D. Turner

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***Trigonometry*, Eighth Edition**
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PREFACE

THIS EIGHTH EDITION of *Trigonometry* preserves the popular format and style of the previous editions. It is a standard right triangle approach to trigonometry, providing a smooth and integrated development of the six trigonometric functions from point-on-the-terminal-side, to right triangle, to circular function definitions. Nearly every section is written so that it can be discussed in a typical class session.

The focus of the text is on understanding the definitions and principles of trigonometry and their applications to problem solving. Exact values of the trigonometric functions are emphasized throughout the text. The clean layout and conversational style encourage students to read the text. Historical vignettes are placed throughout the text to give students an appreciation for the rich history behind trigonometry and the people who contributed to its development.

The text covers all the material usually taught in trigonometry. There is also an appendix that provides a review of algebra, geometry, functions, and inverse functions. The appendix sections can be used as a review of topics with which students may already be familiar, or they can be used to provide instruction for students encountering these concepts for the first time.

Numerous calculator notes are placed throughout the text to help students calculate values when appropriate. As there are many different models of graphing calculators, and each model has its own set of commands, we have tried to avoid overuse of specific key icons or command names.

NEW TO THIS EDITION

Content Changes

The following list describes the major content changes you will see in this eighth edition.

- **APPENDIX:** In response to a number of requests, we have added two new appendix sections providing a substantial review of algebra and geometry. The content of these appendix sections focuses on the specific concepts and skills students will see or need at various points throughout the text.
- **SECTION 1.1:** A proof of the Pythagorean Theorem has been moved out of this section and placed in the appendix.
- **SECTION 7.5:** A general formula for the vector component form of a vector is now given, based on the magnitude of the vector and the angle it makes with the positive x-axis.

NEW EXAMPLES New examples have been added to every chapter to help students gain a better understanding of certain concepts.

NEW OR REVISED EXERCISES Many of the exercises have been revised, and new exercises and application problems have been added in some sections to help students better grasp key concepts, and to help motivate students and stimulate their interest in trigonometry. The most significant additions include the following.

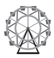
- **SECTION 4.4:** We have added more exercises to this section, giving students the opportunity for additional practice identifying the period, range, and horizontal and vertical translations for the tangent, cotangent, secant, and cosecant functions.
- **SECTION 6.1:** We have increased the number of exercises that require approximating solutions and for which the argument of the trigonometric function involves a horizontal translation.
- **SECTION 6.3:** There are now exercises containing trigonometric functions with arguments in the form $Bx + C$; we have also increased the number of application problems.

EXTENDING THE CONCEPTS More of these problems have been added throughout the text to give students additional challenges and the opportunity to explore certain topics further.

CONTINUING FEATURES

THREE DEFINITIONS All three definitions for the trigonometric functions are contained in the text. The point-on-the-terminal-side definition is contained in Section 1.3; the right triangle definition in Section 2.1; and circular functions are given in Section 3.3.

THEMES There are a number of themes that run throughout the text, and we have clearly marked these themes in the problem sets with appropriate icons. Here is a list of the icons and corresponding themes.

 Ferris Wheels

 Human Cannonball

Navigation

 Sports

CHAPTER INTRODUCTIONS Each chapter opens with an introduction in which a real-world application, historical example, or link between topics is used to stimulate interest in the chapter. Many of these introductions are expanded in the chapter and then carried through to topics found later in the text. Many sections open in a similar fashion.

STUDY SKILLS Study Skills sections, found in the first six chapter openings, help students become organized and efficient with their time.

STUDENT LEARNING OBJECTIVES Each section begins with a list of student learning objectives, which describe the specific, measurable knowledge and skills that students are expected to achieve. Learning objectives help the student identify and focus on the important concepts in each section, and increase the likelihood of their success by establishing clear goals. For instructors, learning objectives can help in organizing class lessons and learning activities, and in creating student assessments.

MATCHED PRACTICE PROBLEMS In every section of this text, each example is paired with a matched practice problem that is similar to the example. These problems give students an opportunity to practice what they have just learned before moving on to the next example. Instructors may want to use them as in class examples or to provide guided practice activities in class. Answers are given in the answers section in the back of the text.

USING TECHNOLOGY *Using Technology* sections throughout the text show how graphing calculator technology can be used to enhance the topics covered. All graphing calculator material is optional, but even if you are not using graphing calculator technology in your classroom, these segments can provide additional insight into the standard trigonometric procedures and problem solving found in the section.

GETTING READY FOR CLASS Located before each problem set, *Getting Ready for Class* sections feature questions that require written responses from students, and which can be answered by reading the preceding section. They are to be done before the students come to class.

CONCEPTS AND VOCABULARY Each problem set begins with a set of questions that focus on grasping the main ideas and understanding the vocabulary and terminology presented in that particular section. Most of these questions are short-answer, but some also include matching or other formats.

GRAPHING CALCULATOR EXERCISES Exercises that require graphing calculators are included in some of the problem sets. These exercises are clearly marked with a special icon and may easily be omitted if you are not using this technology in your classroom. Some of these exercises are investigative in nature, and help prepare students for concepts that are introduced in following sections.

APPLICATIONS Application problems are titled according to subject and indexed for easy reference. We have found that students are more likely to put time and effort into application problems if they do not have to work an overwhelming number of them at one time, and if they work on them every day. For this reason, a few application problems are included toward the end of almost every problem set in the text.

EXTENDING THE CONCEPTS Scattered throughout the text, *Extending the Concepts* problems give students an opportunity to explore certain topics further or take on a more challenging problem.

REVIEW PROBLEMS Beginning with Chapter 2, each problem set contains a few review problems. Where appropriate, the review problems cover material that will be needed in the next section; otherwise, they cover material from the previous chapter. Continual review will help students retain what they have learned in previous chapters and reinforce main ideas.

LEARNING OBJECTIVES ASSESSMENTS Multiple-choice questions are provided at the end of every problem set and are designed to be used in or outside of class to assess student learning. Each question directly corresponds to one of the student learning objectives for that section. Answers to these questions are not available to students, but are provided for instructors in the Instructor's Edition and the Instructor's Solutions Manual. These problems can be especially useful for schools and institutions required to provide documentation and data relating to assessment of student learning outcomes.

GROUP PROJECTS Each chapter concludes with a group project which involves some interesting problem or application that relates to or extends the ideas introduced in the chapter. Many of the projects emphasize the connection of mathematics with other disciplines, or illustrate real-life situations in which trigonometry is used. The projects are designed to be used in class with groups of three or four students each, but the problems could also be given as individual assignments for students wanting an additional challenge.

RESEARCH PROJECTS At least one research project is also offered at the end of each chapter. The research projects ask students to investigate a historical topic or personage that is in some way connected to the material in the chapter, and are intended to promote an appreciation for the rich history behind trigonometry. Students may find the local library or the Internet to be helpful resources in doing their research.

CHAPTER SUMMARIES Each chapter summary lists the new properties and definitions found in the chapter. The margins in the chapter summaries contain examples that illustrate the topics being reviewed.

CHAPTER TESTS Every chapter ends with a chapter test that contains a representative sample of the problems covered in the chapter. The chapter tests were designed to be short enough so that a student may work all the problems in a reasonable amount of time. If you want to reduce the number of problems even further, you can assign just the odd or the even problems. Answers to both odd and even problems for chapter tests are given in the back of the text.

CUMULATIVE TESTS To help students review previous learning and better retain information, three cumulative tests appear in the text. These are similar to the chapter tests, except that the questions pertain to all the sections in the book up to that point. The Cumulative Tests are good resources for students studying for a midterm or final exam. Answers to both odd and even problems for cumulative tests are given in the back of the text.

SUPPLEMENTS TO THE TEXT

For the Instructor:

COMPLETE SOLUTIONS MANUAL (ISBN: 978-1-305-94573-9) Contains all worked-out solutions to the exercises and chapter tests.

TEST BANK (ISBN: 978-1-305-94574-6) Provides multiple test forms per chapter as well as final exams. The tests combine multiple-choice, free-response, and fill-in-the-blank questions for your convenience.

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For the Student

STUDENT SOLUTIONS MANUAL (ISBN: 978-1-305-87786-3) Contains worked-out solutions to all of the odd-numbered exercises in the text, giving students a way to check their answers and ensure that they took the correct steps to arrive at an answer.

TEXT-SPECIFIC DVDs Provides students with visual reinforcement of concepts and explanations given in easy-to-understand terms, with detailed examples and sample problems. A flexible format offers versatility for quickly accessing topics or catering lectures to self-paced, online, or hybrid courses. Closed captioning is provided for the hearing impaired.

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ACKNOWLEDGMENTS

First and foremost, we are thankful to our many loyal users who have helped make this book one of the most popular trigonometry texts on the market. We sincerely appreciate your comments and words of encouragement offered at conferences and by e-mail.

In producing this eighth edition, we relied on the help of many industrious people. Gary Whalen, our Product Manager, and Stacy Green, Senior Content Developer, were instrumental in guiding us through this latest revision. Vernon Boes, Senior Art Director, provided us with the new cover and design. Jennifer Risden, Senior Content Product Manager, helped ensure a smooth passage through the production process. Prashant Das, Senior Project Manager at MPS Limited, did an outstanding job handling the copyediting and composition. His team made our job so much easier. Our accuracy checker, Ann Ostberg, was a great help in ensuring that this new edition has very few errors (hopefully none). Judy Barclay, formerly at Cuesta College, and Ross Rueger, College of the Sequoias, continued in their roles as authors of the solutions manuals. Our thanks go to all these people; this book would not have been possible without them.

Finally, a number of people provided us with suggestions and helpful comments on this revision, including some who performed a review or were asked to give feedback on specific topics. We are grateful to all of the following people for their help with this revision.

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Bradley Stetson, *Schoolcraft College*
Phil Veer, *Johnson County Community College*

Brad Stetson, in particular, deserves special thanks for his thoughts and advice regarding the two new review sections we added in the appendix.

Charles P. McKeague
Mark D. Turner
July, 2015

A SPECIAL NOTE TO THE STUDENT

Trigonometry can be a very enjoyable subject to study. You will find that there are many interesting and useful problems that trigonometry can be used to solve. However, many trigonometry students are apprehensive at first because they are worried they will not understand the topics we cover. When we present a new topic that they do not grasp completely, they think something is wrong with them for not understanding it. On the other hand, some students are excited about the course from the beginning. They are not worried about understanding trigonometry and, in fact, expect to find some topics difficult.

What is the difference between these two types of students?

Those who are excited about the course know from experience (as you do) that a certain amount of confusion is associated with most new topics in mathematics. They don't worry about it, because they also know that the confusion gives way to understanding in the process of reading the text, working problems, and getting their questions answered. If they find a topic difficult, they work as many problems as necessary to grasp the subject. They don't wait for the understanding to come to them; they go out and get it by working lots of problems. In contrast, the students who lack confidence tend to give up when they become confused. Instead of working more problems, they sometimes stop working problems altogether — and that, of course, guarantees that they will remain confused.

If you are worried about this course because you lack confidence in your ability to understand trigonometry, and you want to change the way you feel about mathematics, then look forward to the first topic that causes you some confusion. As soon as that topic comes along, make it your goal to master it, in spite of your apprehension. You will see that each and every topic covered in this course is one you can eventually master, even if your initial introduction to it is accompanied by some confusion. As long as you have passed a college-level intermediate algebra course (or its equivalent), you are ready to take this course.

It also helps a great deal if you make a solid commitment to your trigonometry course. If you are not completely committed to a class, then you will tend to give less than your full effort. Consider this quote from Johann Wolfgang von Goethe's *Faust*:

> *Until one is committed, there is hesitancy, the chance to draw back, always ineffectiveness. Concerning all acts of initiative and creation, there is one elementary truth the ignorance of which kills countless ideas and splendid plans: that the moment one definitely commits oneself, then providence moves too. All sorts of things occur to help one that would never otherwise have occurred. A whole stream of events issues from the decision, raising in one's favor all manner of unforeseen incidents, meetings and material assistance which no man could have dreamed would have come his way. Whatever you can do or dream you can, begin it. Boldness has genius, power and magic in it.*

If you are committed to doing well in trigonometry, the following suggestions will be important to you.

How to Be Successful in Trigonometry

1. **ATTEND ALL CLASS SESSIONS ON TIME.** You cannot know exactly what goes on in class unless you are there. Missing class and then expecting to find out what went on from someone else is not the same as being there yourself.

2. **READ THE TEXT.** This text was written for you! It is best to read beforehand the section that will be covered in class. Reading in advance, even if you do not understand everything you read, helps prepare a foundation for what you will see in class. Also, your instructor may not have time to discuss everything you need to know from a section, so you may need to pick up some things on your own.

3. **WORK PROBLEMS EVERY DAY AND CHECK YOUR ANSWERS.** The key to success in mathematics is working problems. The more problems you work, the better you will become at working them. The answers to the odd-numbered problems are given in the back of the book. When you have finished an assignment, be sure to compare your answers with those in the text. If you have made a mistake, find out what it is, and correct it.

4. **DO IT ON YOUR OWN.** Don't be misled into thinking that someone else's work is your own. Having someone else show you how to work a problem is not the same as working that problem yourself. It is okay to get help when you are stuck. As a matter of fact, it is a good idea. Just be sure you do the work yourself, and that you can work the entire problem correctly on your own later on.

5. **REVIEW EVERY DAY.** After you have finished the problems your instructor has assigned, take another 15 minutes and review a section you have already completed. The more you review, the longer you will retain the material you have learned.

6. **DON'T EXPECT TO UNDERSTAND EVERY NEW TOPIC THE FIRST TIME YOU SEE IT.** Sometimes you will understand everything you are doing, and sometimes you won't. That's just the way things are in mathematics. Expecting to understand each new topic the first time you see it can lead to disappointment and frustration. The process of understanding trigonometry takes time. It requires you to read the text, work problems, and get your questions answered.

7. **SPEND AS MUCH TIME AS IT TAKES FOR YOU TO MASTER THE MATERIAL.** No set formula exists for the exact amount of time you need to spend on trigonometry to master it. You will find out as you go along what is or isn't enough time for you. If you end up spending two or more hours on each section in order to master the material there, then that's how much time it takes; trying to get by with less will not work.

8. **RELAX.** It's probably not as difficult as you think.

1

D. E. SMITH *Without Thales there would not have been a Pythagoras—or such a Pythagoras; and without Pythagoras there would not have been a Plato—or such a Plato.*

The Six Trigonometric Functions

Introduction

The history of mathematics is a spiral of knowledge passed down from one generation to another. Each person in the history of mathematics is connected to the others along this spiral. In Italy, around 500 B.C., the Pythagoreans discovered a relationship between the sides of any right triangle. That discovery, known as the Pythagorean Theorem, is the foundation on which the Spiral of Roots shown in Figure 1 is built. The Spiral of Roots gives us a way to visualize square roots of positive integers.

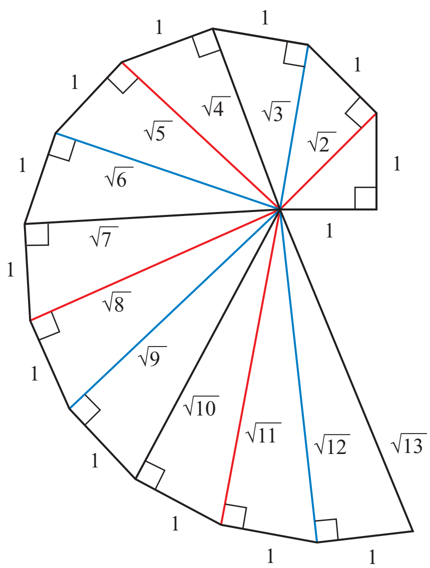


Figure 1

In Problem Set 1.1, you will have a chance to construct the Spiral of Roots yourself.

Study Skills 1

At the beginning of the first few chapters of this text you will find a Study Skills section in which we list the skills that are necessary for success in trigonometry. If you have just completed an algebra class successfully, you have acquired most of these skills. If it has been some time since you have taken a math class, you must pay attention to the sections on study skills.

Here is a list of things you can do to develop effective study skills.

1 **Put Yourself on a Schedule** The general rule is that you spend two hours on homework for every hour you are in class. Make a schedule for yourself, setting aside at least six hours a week to work on trigonometry. Once you make the schedule, stick to it. Don't just complete your assignments and then stop. Use all the time you have set aside. If you complete an assignment and have time left over, read the next section in the text and work more problems. As the course progresses you may find that six hours a week is not enough time for you to master the material in this course. If it takes you longer than that to reach your goals for this course, then that's how much time it takes. Trying to get by with less will not work.

2 **Find Your Mistakes and Correct Them** There is more to studying trigonometry than just working problems. You must always check your answers with those in the back of the text. When you have made a mistake, find out what it is and correct it. Making mistakes is part of the process of learning mathematics. The key to discovering what you do not understand can be found by correcting your mistakes.

3 **Imitate Success** Your work should look like the work you see in this text and the work your instructor shows. The steps shown in solving problems in this text were written by someone who has been successful in mathematics. The same is true of your instructor. Your work should imitate the work of people who have been successful in mathematics.

4 **Memorize Definitions and Identities** You may think that memorization is not necessary if you understand a topic you are studying. In trigonometry, memorization is especially important. In this chapter, you will be presented with the definition of the six trigonometric functions that you will use throughout your study of trigonometry. We have seen many bright students struggle with trigonometry simply because they did not memorize the definitions and identities when they were first presented. ■

SECTION 1.1 Angles, Degrees, and Special Triangles

Learning Objectives

1 Compute the complement and supplement of an angle.
2 Use the Pythagorean Theorem to find the third side of a right triangle.
3 Find the other two sides of a $30°–60°–90°$ or $45°–45°–90°$ triangle given one side.
4 Solve a real-life problem using the special triangle relationships.

TABLE 1
From the Trail Map for the Northstar California Ski Resort

Lift Information		
Lift	**Vertical Rise (ft)**	**Length (ft)**
Big Springs Express	480	4,100
Lookout Link	960	4,330
Comstock Express	1,250	5,900
Rendezvous	650	2,900

Introduction

Table 1 is taken from the trail map at the Northstar California Ski Resort in Lake Tahoe, California. The table gives the length of some of the chair lifts at Northstar, along with the change in elevation from the beginning of the lift to the end of the lift.

Right triangles are good mathematical models for chair lifts. In this section we review some important items from geometry, including right triangles. Let's begin by looking at some of the terminology associated with angles.

Angles in General

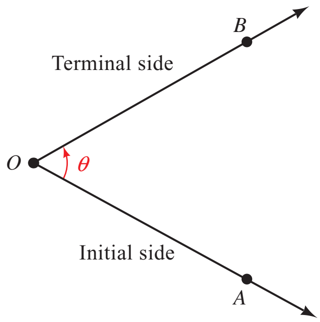


Figure 1

An angle is formed by two rays with the same end point. The common end point is called the *vertex* of the angle, and the rays are called the *sides* of the angle.

In Figure 1 the vertex of angle θ (theta) is labeled O, and A and B are points on each side of θ. Angle θ can also be denoted by AOB, where the letter associated with the vertex is written between the letters associated with the points on each side.

We can think of θ as having been formed by rotating side OA about the vertex to side OB. In this case, we call side OA the *initial side* of θ and side OB the *terminal side* of θ.

When the rotation from the initial side to the terminal side takes place in a counterclockwise direction, the angle formed is considered a *positive angle.* If the rotation is in a clockwise direction, the angle formed is a *negative angle* (Figure 2).

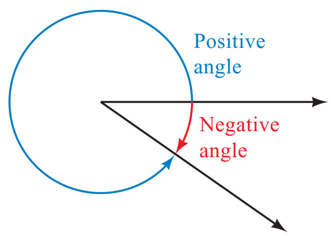
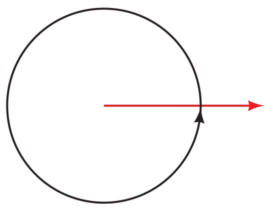


Figure 2

Degree Measure


One complete revolution = 360°
Figure 3

One way to measure the size of an angle is with degree measure. The angle formed by rotating a ray through one complete revolution has a measure of 360 degrees, written 360° (Figure 3).

One degree (1°), then, is 1/360 of a full rotation. Likewise, 180° is one-half of a full rotation, and 90° is half of that (or a quarter of a rotation). Angles that measure 90° are called *right angles,* while angles that measure 180° are called *straight* *angles.* Angles that measure between 0° and 90° are called *acute angles,* while angles that measure between 90° and 180° are called *obtuse angles* (see Figure 4).

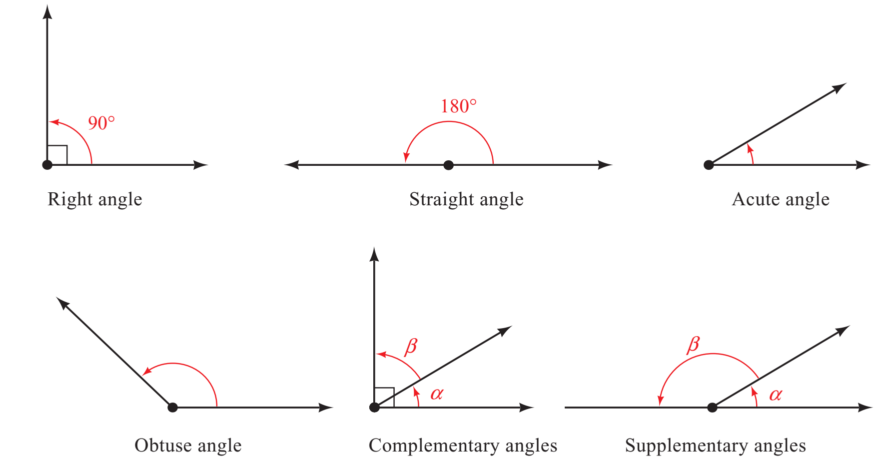


Figure 4

If two angles have a sum of 90°, then they are called *complementary angles,* and we say each is the *complement* of the other. Two angles with a sum of 180° are called *supplementary angles.*

NOTE To be precise, we should say "two angles, the sum of the measures of which is 180°, are called supplementary angles" because there is a difference between an angle and its measure. However, in this text, we will not always draw the distinction between an angle and its measure. Many times we will refer to "angle θ" when we actually mean "the measure of angle θ."

NOTE The little square by the vertex of the right angle in Figure 4 is used to indicate that the angle is a right angle. You will see this symbol often in the text.

PROBLEM 1
Give the complement and supplement of each angle.
a. 25°
b. 118°
c. β

EXAMPLE 1 Give the complement and the supplement of each angle.

a. 40° **b.** 110° **c.** θ

SOLUTION

a. The complement of 40° is 50° since 40° + 50° = 90°.
The supplement of 40° is 140° since 40° + 140° = 180°.

b. The complement of 110° is −20° since 110° + (−20°) = 90°.
The supplement of 110° is 70° since 110° + 70° = 180°.

c. The complement of θ is $90° - \theta$ since $\theta + (90° - \theta) = 90°$.
The supplement of θ is $180° - \theta$ since $\theta + (180° - \theta) = 180°$.

Triangles

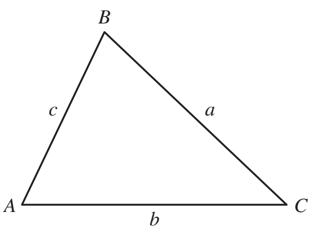


Figure 5

A triangle is a three-sided polygon. Every triangle has three sides and three angles. We denote the angles (or vertices) with uppercase letters and the lengths of the sides with lowercase letters, as shown in Figure 5. It is standard practice in mathematics to label the sides and angles so that a is opposite A, b is opposite B, and c is opposite C.

There are different types of triangles that are named according to the relative lengths of their sides or angles (Figure 6). In an *equilateral triangle,* all three sides are of equal length and all three angles are equal. An *isosceles triangle* has two equal sides and two equal angles. If all the sides and angles are different, the triangle is called *scalene*. In an *acute triangle,* all three angles are acute. An *obtuse triangle* has exactly one obtuse angle, and a *right triangle* has one right angle.

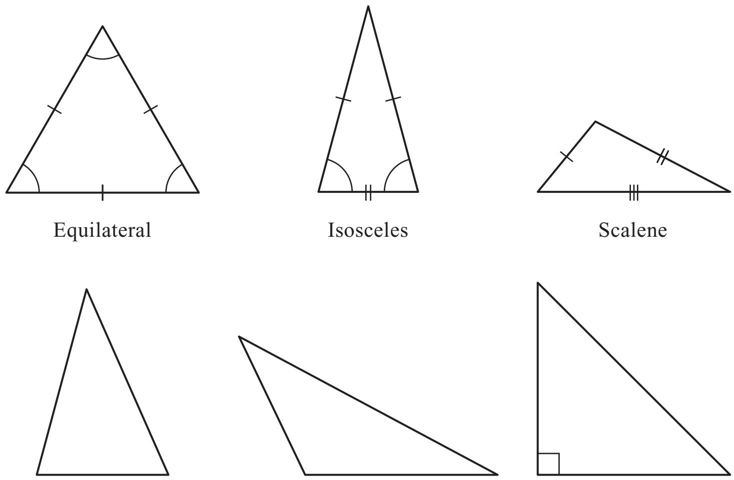


Figure 6

Special Triangles

As we will see throughout this text, right triangles are very important to the study of trigonometry. In every right triangle, the longest side is called the *hypotenuse,* and it is always opposite the right angle. The other two sides are called the *legs* of the right triangle. Because the sum of the angles in any triangle is 180°, the other two angles in a right triangle must be complementary, acute angles. The Pythagorean Theorem that we mentioned in the introduction to this chapter gives us the relationship that exists among the sides of a right triangle.

Pythagorean Theorem

In any right triangle, the square of the length of the longest side (called the hypotenuse) is equal to the sum of the squares of the lengths of the other two sides (called legs).

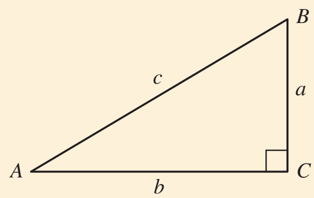


Figure 7

If $C = 90°$,
then $c^2 = a^2 + b^2$.

There are many ways to prove the Pythagorean Theorem. We offer one proof in Appendix A.2. The Group Project at the end of this chapter introduces several more of these ways.

PROBLEM 2
Solve for x in Figure 9.

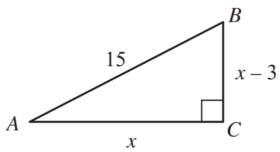


Figure 9

EXAMPLE 2 Solve for x in the right triangle in Figure 8.

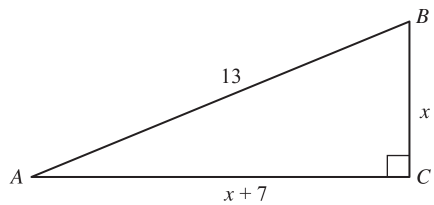


Figure 8

SOLUTION Applying the Pythagorean Theorem gives us a quadratic equation to solve.

$$
\begin{aligned}
(x + 7)^2 + x^2 &= 13^2 \\
x^2 + 14x + 49 + x^2 &= 169 && \text{Expand } (x+7)^2 \text{ and } 13^2 \\
2x^2 + 14x + 49 &= 169 && \text{Combine similar terms} \\
2x^2 + 14x - 120 &= 0 && \text{Add } -169 \text{ to both sides} \\
x^2 + 7x - 60 &= 0 && \text{Divide both sides by 2} \\
(x - 5)(x + 12) &= 0 && \text{Factor the left side} \\
x - 5 = 0 \quad \text{or} \quad x + 12 &= 0 && \text{Set each factor to 0} \\
x = 5 \quad \text{or} \quad x &= -12
\end{aligned}
$$

Our only solution is $x = 5$. We cannot use $x = -12$ because x is the length of a side of triangle ABC and therefore cannot be negative. ■

NOTE The lengths of the sides of the triangle in Example 2 are 5, 12, and 13. Whenever the three sides in a right triangle are natural numbers, those three numbers are called a *Pythagorean triple.*

PROBLEM 3
Repeat Example 3 for the Big Springs Express chair lift.

EXAMPLE 3 Table 1 in the introduction to this section gives the vertical rise of the Comstock Express chair lift as 1,250 feet and the length of the chair lift as 5,900 feet. To the nearest foot, find the horizontal distance covered by a person riding this lift.

SOLUTION Figure 10 is a model of the Comstock Express chair lift. A rider gets on the lift at point A and exits at point B. The length of the lift is AB.

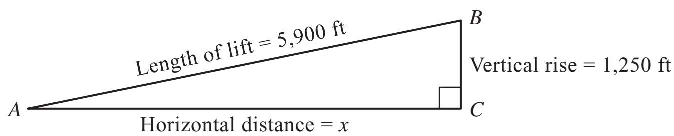


Figure 10

To find the horizontal distance covered by a person riding the chair lift, we use the Pythagorean Theorem:

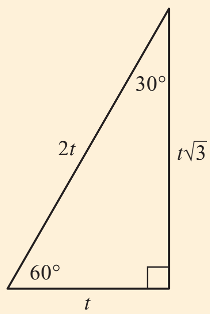
$5{,}900^2 = x^2 + 1{,}250^2$	Pythagorean Theorem
$34{,}810{,}000 = x^2 + 1{,}562{,}500$	Simplify squares
$x^2 = 34{,}810{,}000 - 1{,}562{,}500$	Solve for x^2
$x^2 = 33{,}247{,}500$	Simplify the right side
$x = \sqrt{33{,}247{,}500}$	
$x = 5{,}766$ ft	To the nearest foot

A rider getting on the lift at point A and riding to point B will cover a horizontal distance of approximately 5,766 feet.

Before leaving the Pythagorean Theorem, we should mention something about Pythagoras and his followers, the Pythagoreans. They established themselves as a secret society around the year 540 B.C. The Pythagoreans kept no written record of their work; everything was handed down by spoken word. Their influence was not only in mathematics, but also in religion, science, medicine, and music. Among other things, they discovered the correlation between musical notes and the reciprocals of counting numbers, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and so on. In their daily lives they followed strict dietary and moral rules to achieve a higher rank in future lives. The British philosopher Bertrand Russell has referred to Pythagoras as "intellectually one of the most important men that ever lived."

The 30°–60°–90° Triangle

In any right triangle in which the two acute angles are 30° and 60°, the longest side (the hypotenuse) is always twice the shortest side (the side opposite the 30° angle), and the side of medium length (the side opposite the 60° angle) is always $\sqrt{3}$ times the shortest side (Figure 11).



30°–60°–90°

Figure 11

NOTE The shortest side t is opposite the smallest angle 30°. The longest side $2t$ is opposite the largest angle 90°.

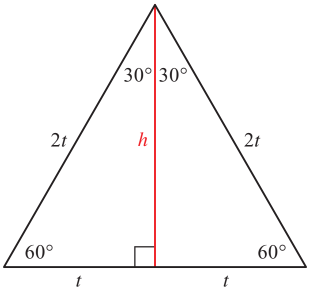


Figure 12

To verify the relationship between the sides in this triangle, we draw an equilateral triangle (one in which all three sides are equal) and label half the base with t (Figure 12).

The altitude h (the colored line) bisects the base. We have two 30°–60°–90° triangles. The longest side in each is $2t$. We find that h is $t\sqrt{3}$ by applying the Pythagorean Theorem.

$$\begin{aligned} t^2 + h^2 &= (2t)^2 \\ h &= \sqrt{4t^2 - t^2} \\ &= \sqrt{3t^2} \\ &= t\sqrt{3} \end{aligned}$$

PROBLEM 4
If the longest side of a 30°–60°–90° triangle is 14, find the lengths of the other two sides.

EXAMPLE 4 If the shortest side of a 30°–60°–90° triangle is 5, find the other two sides.

SOLUTION The longest side is 10 (twice the shortest side), and the side opposite the 60° angle is $5\sqrt{3}$ (Figure 13).

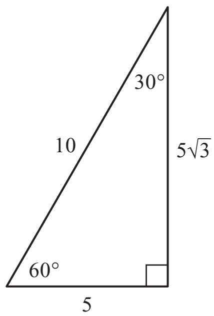


Figure 13

PROBLEM 5
A ladder is leaning against a wall. The bottom of the ladder makes an angle of 60° with the ground and is 3 feet from the base of the wall. How long is the ladder and how high up the wall does it reach?

EXAMPLE 5 A ladder is leaning against a wall. The top of the ladder is 4 feet above the ground and the bottom of the ladder makes an angle of 60° with the ground (Figure 14). How long is the ladder, and how far from the wall is the bottom of the ladder?

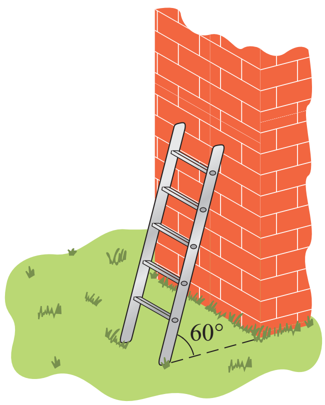


Figure 14

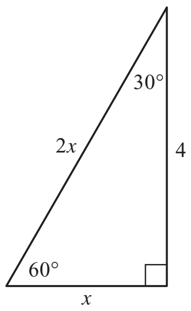


Figure 15

SOLUTION The triangle formed by the ladder, the wall, and the ground is a 30°–60°–90° triangle. If we let x represent the distance from the bottom of the ladder to the wall, then the length of the ladder can be represented by $2x$. The distance from the top of the ladder to the ground is $x\sqrt{3}$, since it is opposite the 60° angle (Figure 15). It is also given as 4 feet. Therefore,

$$x\sqrt{3} = 4$$
$$x = \frac{4}{\sqrt{3}}$$
$$= \frac{4\sqrt{3}}{3}$$

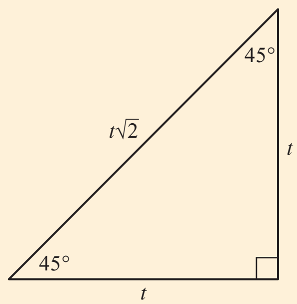
Rationalize the denominator by multiplying the numerator and denominator by $\sqrt{3}$.

The distance from the bottom of the ladder to the wall, x, is $4\sqrt{3}/3$ feet, so the length of the ladder, $2x$, must be $8\sqrt{3}/3$ feet. Note that these lengths are given in exact values. If we want a decimal approximation for them, we can replace $\sqrt{3}$ with 1.732 to obtain

$$\frac{4\sqrt{3}}{3} \approx \frac{4(1.732)}{3} = 2.309 \text{ ft}$$
$$\frac{8\sqrt{3}}{3} \approx \frac{8(1.732)}{3} = 4.619 \text{ ft}$$

The 45°–45°–90° Triangle

If the two acute angles in a right triangle are both 45°, then the two shorter sides (the legs) are equal and the longest side (the hypotenuse) is $\sqrt{2}$ times as long as the shorter sides. That is, if the shorter sides are of length t, then the longest side has length $t\sqrt{2}$ (Figure 16).



45°–45°–90°

Figure 16

To verify this relationship, we simply note that if the two acute angles are equal, then the sides opposite them are also equal. We apply the Pythagorean Theorem to find the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{t^2 + t^2}$$
$$= \sqrt{2t^2}$$
$$= t\sqrt{2}$$